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EFFECT OF REFLECTED STRESS WAVES IN FINITE SPECIMENS ON THE DYNAMIC STRESS INTENSITY FACTOR OF A PROPAGATING MODE III CRACK

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Abstract—The effect of reflected stress waves in a finite plate on the dynamic stress intensity factor of a propagating mode III crack has been considered. Kostrov's analysis enables the effect of reflections from one specimen boundary (the cracked surface) to be taken into account, but cannot be applied directly to include reflections from the other (uncracked) surface. However, solutions may be obtained, if Kostrov's analysis is used in a modified form and the stress waves reflected from the uncracked surface of the specimen are treated separately as incident waves employing a related analysis due to Achenbach. The method of solution has been applied to a simple example. In this case the effect of the arrival of the first stress waves reflected from the uncracked surface is to cause a small gradual increase in the dynamic stress intensity factor.

1. INTRODUCTION

One of the major difficulties in the solution of problems of dynamic crack propagation is the effect of the reflection or diffraction of stress waves at the specimen boundaries or at the crack tips. Kostrov's analysis[1] for the dynamic propagation of a mode III crack of finite length enables the effect of the diffraction of stress waves at the opposite crack tip to be included in the calculation of the stress ahead of the crack. Since in anti-plane strain a free surface reflects the displacement field, this method of solution may also be applied to an edge crack and the effect of reflections of stress waves at the cracked surface may be taken into account. Examples of the application of Kostrov's method of solution are given in Refs. [2–7].

Of considerable practical interest, e.g. in a loss of coolant accident (LOCA) in a pressurized water reactor (PWR) is whether the crack will propagate right through the thickness of the body, or whether it will arrest before this. Determination of the answer to this requires knowledge of the effect of reflections from both surfaces of the body. Some insight into this problem may be obtained by considering the anti-plane strain situation. In the work reported here it is shown that by superposition of Kostrov's problem for dynamic crack propagation, and a related problem considered by Achenbach[8–10] see also [11] for the effect of incident stress waves on a crack, it is possible to take into account the effect of the reflection of stress waves from both surfaces of a specimen during the propagation of a mode III crack.

2. CRACK IN AN INFINITE SPECIMEN

The aim of this section is to review Kostrov's method of solution and to show why it cannot be applied directly to the problem of a crack in a finite body, when reflections from both surfaces must be taken into account.

The problem considered by Kostrov[1] is that of a crack of length $2a$ in an infinite plate. The crack lies in the $y = 0$ plane, with its centre at $x = 0$, and starts to propagate at time $t = 0$. It is convenient to work in the characteristic coordinates of the system.

$$\eta = \frac{1}{\sqrt{2}}(s + x) \tag{1}$$

$$\xi = \frac{1}{\sqrt{2}}(s - x) \tag{2}$$

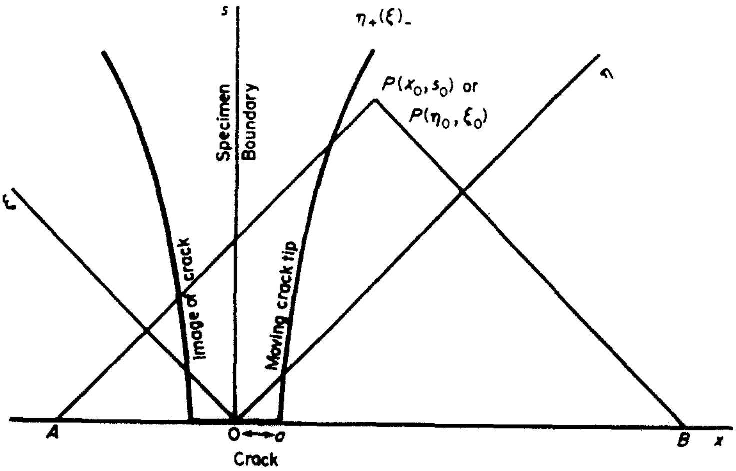

Fig. 1. Representation in the η-ξ plane of the motion of an edge crack in a semi-infinite plate (Kostrov's problem), showing the trajectories of the moving crack tip and the image of the crack tip.

as shown in Fig. 1, where $s = ct$ where c is the velocity of shear waves. From Volterra's theorem Kostrov[1] gives the change in displacement at some point $P(x_0, s_0)$ in the plane of the crack ($y = 0$) at time t_0 as

$$w'(x_0, t_0) = \frac{c}{\mu\pi} \int\int \frac{\tau'(x, s)\,\mathrm{d}x\,\mathrm{d}s}{[(s_0 - s)^2 - (x_0 - x)^2]^{1/2}} \tag{3}$$

where τ' is the change in the τ_{yz} stress from its initial value before the start of crack propagation, μ is the shear modulus and the area of integration is bounded by the lines

$$(x_0 - x) = \pm(s_0 - s) \tag{4}$$

i.e. integration is over that part of the triangle APB of Fig. 1 over which there is a change in stress. In terms of the characteristic coordinates this gives

$$w'(\eta_0, \xi_0) = \frac{c}{\mu\pi} \int_{-\eta_0}^{\xi_0} \frac{\mathrm{d}\xi}{(\xi_0 - \xi)^{1/2}} \int_{-\xi_0}^{\eta_0} \frac{\mathrm{d}\eta}{(\eta_0 - \eta)^{1/2}} [\tau'(\xi, \eta)] \tag{5}$$

where in general the lower limits of integration are $-\xi_0$ and $-\eta_0$, but the area of integration need only include those parts of the $\eta - \xi$ plane where there is a change of stress. The displacement is zero ahead of the crack, and thus provided $P(\eta_0, \xi_0)$ lies ahead of the crack tip, this integral may be set equal to zero. In Kostrov's analysis this equation is further reduced to

$$0 = \int_{-\xi_0}^{\eta_0} \frac{\tau'(\eta, \xi_0)}{(\eta_0 - \eta)^{1/2}} \mathrm{d}\eta \tag{6}$$

The reason why this reduction may be made for the infinite plate system (although not given in Kostrov's analysis is that the change in the displacement $w'(\eta_0, \xi')$ (as given by eqn 5) is zero for all points $P'(\eta_0, \xi')$ for $\xi' \le \xi_0$, i.e. for all points along the line PB of Fig. 1. This then requires that the single integral of eqn (6) is equal to zero. However, if as in the case to be considered here, the displacement is not zero for all points along the line PB, because this then includes the reflected image of the crack, eqn (6) is no longer valid and Kostrov's analysis cannot be applied directly.

In the case of an infinite plate the integral of eqn (6) may be split into two parts—that ahead of the crack tip at $\eta_+(\xi)$ and that behind it, giving

$$\int_{\eta_+(\xi_0)}^{\eta_0} \frac{\tau'(\eta, \xi_0)}{(\eta_0 - \eta)^{1/2}} \mathrm{d}\eta = -\int_{-\xi_0}^{\eta_+(\xi_0)} \frac{\tau'(\eta, \xi_0)}{(\eta_0 - \eta)^{1/2}} \mathrm{d}\eta. \tag{7}$$

Kostrov has transformed this using Abel's integral to give the change in the stress at some point ahead of the crack tip as

$$\tau'(\eta_0, \xi_0) = -\frac{1}{\pi[\eta_0 - \eta_+(\xi_0)]^{1/2}} \int_{-\xi_0}^{\eta_+(\xi_0)} \frac{\tau'(\eta, \xi_0)[\eta_+(\xi_0) - \eta]^{1/2}}{(\eta_0 - \eta)} \mathrm{d}\eta. \tag{8}$$

In the limit of small distances ahead of the crack tip this gives the stress intensity factor as

$$K = (1 - v/c)^{1/2} \left(\frac{2}{\pi}\right)^{1/2} \int_{-\xi_0}^{\eta_+(\xi)} \frac{-\tau'(\eta, \xi_0)}{[\eta_+(\xi_0) - \eta]^{1/2}} \mathrm{d}\eta. \tag{9}$$

where v is the instantaneous crack velocity. For short times, for $\xi_0 < a\sqrt{2}$ (where $2a$ is the initial crack length) so that the line $\xi = \xi_0$ does not include the opposite crack tip, the result is identical with that found independently by Eshelby[12], and $\tau'(\eta, \xi_0)$ may be identified with the stress ahead of the crack tip prior to the start of crack propagation. For slightly longer times, when $\xi_0 > a\sqrt{2}$, this method of solution must first be applied to the opposite crack tip to determine the change in stress ahead of this and repeated use of this technique is required as the time increases and more reflections have to be taken into account.

3. CRACK PROPAGATION IN A FINITE SPECIMEN

In this section it is shown how Kostrov's method of solution may be extended to include the effect of reflections from the uncracked surface ahead of the crack tip in addition to those from the crack surface behind the crack tip. Burridge and Halliday[7] have used Kostrov's method of solution to consider the interaction of an imbedded crack in a semi-infinite body with the uncracked surface ahead of the crack tip. However, since reflections are considered only for determining the stress after the crack has broken through to the surface, the problem, as far as reflections are concerned is equivalent to that for an edge crack in a semi-infinite body. Here a more general problem is considered.

A mode III edge crack of length a in a specimen of finite width w is shown schematically in Fig. 2(a). Since the free surfaces act as reflectors, the problem is equivalent to an infinite planar array of cracks of length $2a$ and periodicity $2w$. The situation is shown in the $\eta - \xi$ plane in Fig. 2(b), where for convenience the origin has been moved to the uncracked surface and where

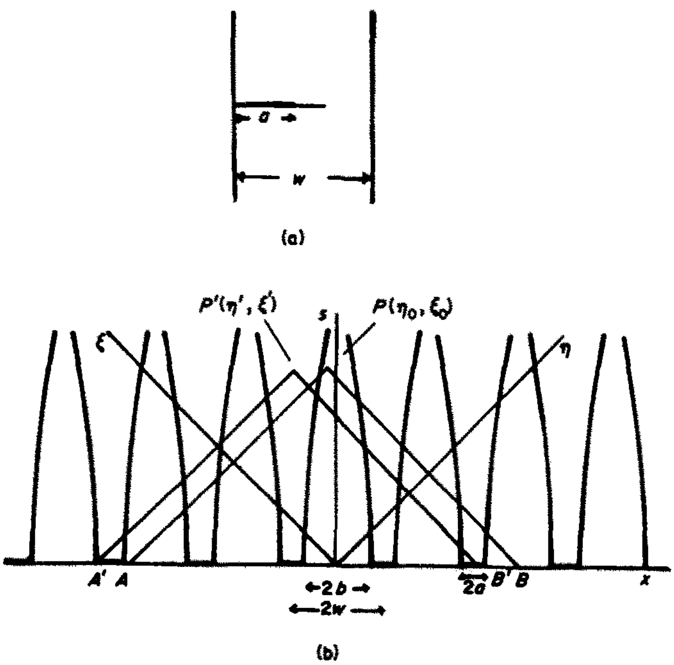

Fig. 2. Motion of an edge crack of initial length a in a plate of finite width w. (a) Specimen geometry. (b) Representation in the η–ξ plane.

$b = (w - a)$ is the length of the uncracked ligament. As before for a point $P\ (\eta_0, \xi_0)$ ahead of the crack tip, the change in the displacement is zero and thus the double integral of eqn (5) may be set equal to zero. However, for $\xi_0 > b/\sqrt{2}$, not all points along the line $\eta = \eta_0$ for $\xi < \xi_0$ correspond to positions where the change in displacement is zero, since parts of this line now fall on the reflected images of the crack. Thus in this case the double integral of eqn (5) cannot be reduced directly to the single integral of eqn (6). However, Kostrov's method of solution may still be used provided the effects of the different stress waves are considered separately. The wave system is shown schematically in Fig. 3. Initially there is only the wave emitted from the propagating crack tip (Fig. 3a). This is then reflected at the specimen boundary so that there is also the reflected wave (Fig. 3b). This reflected wave is then diffracted at the crack tip (Fig. 3c). The changes in stress due to these three waves may be treated separately to give the total change in stress as

$$\tau' = \tau_1 + \tau_2 + \tau_3. \tag{10}$$

Here τ_1 is the change in stress due to waves emitted by the propagating crack, and is the result given by eqn (8), i.e.

$$\tau_1 = -\frac{1}{\pi(\eta_0 - \eta_r)^{1/2}} \int_{-\xi_0}^{\eta_r} \frac{\tau'(\eta, \xi_0)(\eta_r - \eta)^{1/2}}{(\eta_0 - \eta)} \, \mathrm{d}\eta \tag{11}$$

where $\eta_r(\xi_0)$ describes the position of the (real) crack tip. This change is stress may be due just to changes in stress over the length of the crack, or it may already include the effect of stress waves reflected at the cracked surface of the specimen, and calculated by repeated use of

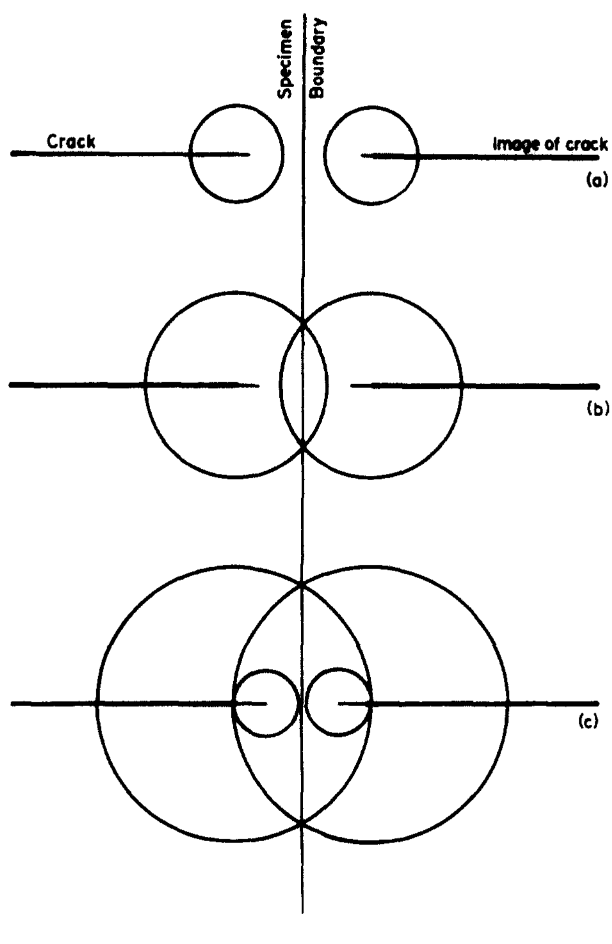

Fig. 3. Effect of reflected and diffracted stress waves with increasing time.

Kostrov's method. The second term τ_2 in the change in stress is due to the reflection of this first wave at the uncracked surface of the specimen. As with τ_1 this may include the effect of reflections at the cracked surface. This is calculated in the same way as τ_1, and is given by

$$\tau_2 = -\frac{1}{\pi(\xi_0 - \xi_1)^{1/2}} \int_{-\eta_0}^{\xi_i} \frac{\tau'(\eta_0, \xi)(\xi_i - \xi)^{1/2}}{(\xi_0 - \xi)} \mathrm{d}\xi. \tag{12}$$

where $\xi_i(\eta_0)$ describes the position of the image of the crack tip reflected in the uncracked surface of the specimen. This second term τ_2 is non-singular at the (real) crack tip and does not contribute directly to the stress intensity factor (see below). The third term τ_3 is due to the diffraction of this second (reflected) wave at the crack tip. Following Achenbach's solution[6–8] for the diffraction of an incident wave by a crack, the τ_{yz} stress along the crack surface must be zero, so that the effect of the diffracted wave is to produce a stress to annul the stress τ_2 that would have been produced by the incident reflected wave along the crack surfaces. Thus the diffracted wave gives a stress

$$\tau^*(\eta, \xi_0) = -\tau_2(\eta, \xi_0) \tag{13}$$

along the crack surfaces. The stress $\tau_2(\eta, \xi_0)$ is the stress that would be given by eqn (12) for a point (η, ξ_0) behind the (real) crack tip. This then gives the third contribution to the change in stress as

$$\tau_3 = +\frac{1}{\pi(\eta_0 - \eta_r)^{1/2}} \int_{\eta^*}^{\eta_r} \frac{\tau_2(\eta, \xi_0)(\eta_r - \eta)^{1/2}}{(\eta_0 - \eta_r)} \mathrm{d}\eta \tag{14}$$

where following Achenbach's analysis for an incident wave[7, 8] the lower limit of integration is given by

$$\begin{aligned} \eta^* &= b/\sqrt{2}, \quad \xi_0 < (2a+b)\sqrt{2} \\ \eta^* &= \eta_l(\xi_0), \quad \xi_0 > (2a+b)\sqrt{2} \end{aligned} \tag{15}$$

where $\eta_l(\xi_0)$ describes the position of the image of the crack tip reflected in the cracked surface of the specimen. As in Achenbach's analysis the integration does not extend beyond the position of the reflected image of the crack tip, since the diffracted wave does not annul the stress due to the reflected wave outside this region. In the limit of small distances ahead of the crack tip these equations give the dynamic stress intensity factor as

$$K_d = (1 - v/c)^{1/2}\left(\frac{2}{\pi}\right)^{1/2}\left[-\int_{-\xi_0}^{\eta_r} \frac{\tau'(\eta, \xi_0)}{(\eta_r - \eta)^{1/2}} \mathrm{d}\eta + \int_{\eta^*}^{\eta_r} \frac{\tau_2(\eta, \xi_0)}{(\eta_r - \eta)^{1/2}} \mathrm{d}\eta\right] \tag{16}$$

where $(1 - v/c)^{1/2}$ is the velocity dependent term of eqn (9).

This superposition approach may be extended to enable a general expression to be written to cover the effect of any number of reflections, although as in Kostrov's analysis repeated use of the equations is required for the evaluation of the change in stress. As shown above, the change in stress ahead of the crack tip at a point $P(\eta_0, \xi_0)$ is given by integrating along the lines $\xi = \xi_0$ and $\eta = \eta_0$ (i.e. along the lines ΔP and AB of Fig. 2b), provided that the changes in stress over the length of the crack (e.g. τ^* oe eqn 13) have already been calculated. In general, by direct extension of Kostrov's and Achenbach's techniques to include the effect of the additional reflections, the change in the stress ahead of the crack tip is then given by

$$\begin{aligned} \tau'(\eta_0, \xi_0) = &-\frac{1}{\pi(\eta_0 - \eta_r)^{1/2}} \int^{\eta_r} \frac{\tau'(\eta, \xi_0)(\eta_r - \eta)^{1/2}}{(\eta_0 - \eta)} \mathrm{d}\eta \\ &-\frac{1}{\pi(\xi_0 - \xi_i)^{1/2}} \int^{\xi_i} \frac{\tau'(\eta_0, \xi)(\xi_i - \xi)^{1/2}}{(\xi_0 - \xi)} \mathrm{d}\xi \end{aligned} \tag{17}$$

where $\eta_r(\xi)$ and $\xi_i(\eta)$ describe the motion of the crack tips or images of the crack tip to the left and right of $P(\eta_0, \xi_0)$ respectively (see Fig. 2b). To enable this to be evaluated, expressions are also required for the changes in the stress along the crack and its images. There are two contributions to this change in stress—that due to the unloading of the stress $p(x)$ ahead of the original crack and its images, and that which is required in Achenbach's analysis to annul the stress due to incident waves. Thus on application of eqn (7) to evaluate the second type of stress, this gives the change in stress for a point $P'(\eta', \xi')$ on the crack or on the image of the crack as

$$\begin{aligned}\tau'(\eta', \xi') &= -p(x)\\ &+\frac{1}{\pi(\eta'-\eta_R)^{1/2}}\int^{\eta_R}\frac{\tau'(\eta, \xi')(\eta_R-\eta)^{1/2}}{(\eta'-\eta)}\,\mathrm{d}\eta\\ &+\frac{1}{\pi(\xi'-\xi_i)^{1/2}}\int^{\xi_i}\frac{\tau'(\eta', \xi)(\xi_i-\xi)^{1/2}}{(\xi'-\xi)}\,\mathrm{d}\xi\end{aligned} \tag{18}$$

where $x=(\eta-\xi)/\sqrt{2}$, and $p(x)$ is the original stress ahead of the crack (images). The function $\eta_R(\xi)$ describes the motion of the right hand tip of the crack (image) to the left of the crack (image) in question and $\xi_i(\eta)$ the motion of the left hand tip of the crack (image) to the right, as is shown in Fig. 2(b). Repeated use of eqns (17) and (18) then enables the stress to be determined for any number of reflections. In the case of an infinite or semi-infinite plate the second integral of eqn (17) and both integrals of eqn (18) disappear, and the equations reduce to those of Kostrov's original analysis.

4. EFFECT OF THE FIRST REFLECTED STRESS WAVE

The effect of the first stress wave reflected from the uncracked edge of the specimen and diffracted from the crack tip may be taken into account in a fairly straight forward manner, as is shown in the example presented here. In general the stress a distance u ahead of a crack in the plane of the crack may be written as

$$p(u)=\frac{1}{(2\pi)^{1/2}}\frac{1}{u^{1/2}}[A_0+A_1u+A_2u^2+\cdots] \tag{19}$$

where the first term determines the stress intensity factor. This stress need only be defined over the length over which it is integrated to determine the dynamic stress intensity factor, i.e. over the length the crack propagates. For simplicitly in the example taken here only the first two terms in this stress will be considered. Further the velocity of crack propagation will be taken as constant. It is also assumed that the initial crack extends a substantial distance across the specimen so that for the first part of crack propagation only reflections from the uncracked surface of the specimen and not those from the cracked surface need be considered. The situation in the $\eta-\xi$ plane is shown in Fig. 4. The motion of the crack tip is then described by

$$\eta_r(\xi)=\frac{1}{(1-v/c)}[(1+v/c)\xi-b\sqrt{2}] \tag{20}$$

and the motion of its image reflected in the uncracked surface is described by

$$\xi_i(\eta)=\frac{1}{(1-v/c)}[(1+v/c)\eta-b\sqrt{2}] \tag{21}$$

The effect of the different terms in $p(u)$ for the stress ahead of the original crack tip may be considered separately, so that in the first instance the situation considered (as in [6]) is that where

$$p(x)=\frac{1}{(2\pi)^{1/2}}\frac{A_0}{u^{1/2}}=\frac{K_0}{(2\pi u)^{1/2}} \tag{22}$$

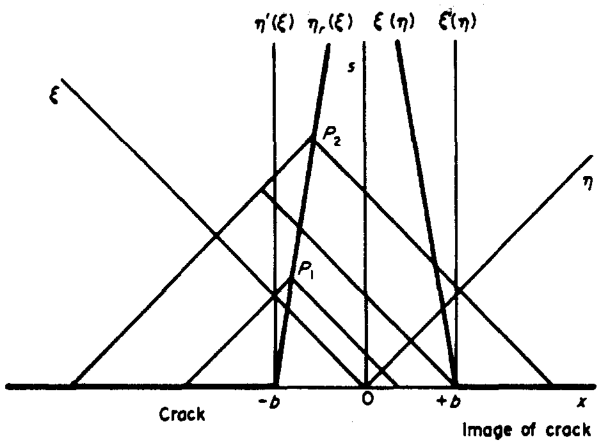

Fig. 4. Diagram for the calculation of the effect of the first stress waves reflected from the uncracked surface of the specimen. P_1—before the arrival of the reflected stress waves. P_2—after the arrival of the reflected stress waves.

where K_0 is the original static stress intensity factor before the start of crack propagation. As shown in[6], the stress intensity factor is then constant until the arrival at the crack tip of the first reflected wave. The stress intensity factor then changes in response to the effect of the reflected and diffracted waves and the effect of the diffracted wave may readily be seen.

Following Kostrov's analysis (see eqn 9) the stress intensity factor before the time of arrival of the first reflected stress wave, i.e. before the time

$$t_1 = \frac{b}{c}\left(\frac{2}{1+v/c}\right) \tag{23}$$

is given by

$$K_d = (1 - v/c)^{1/2}\left(\frac{2}{\pi}\right)^{1/2}\int_{\eta'}^{\eta_r}\frac{K_0}{(2\pi)^{1/2}}\frac{1}{(\eta-\eta')^{1/2}(\eta_r-\eta)^{1/2}}\,\mathrm{d}\eta \tag{24}$$

where $\eta_r(\xi_0)$ is the instantaneous position of the crack tip (point P_1 of Fig. 4) and $\eta'(\xi_0)$ is the original position of the crack tip along the line $\xi = \xi_0$, i.e. $\eta' = \xi_0 - b\sqrt{2}$. Integration of eqn (24) gives simply

$$K_d = (1 - v/c)^{1/2}K_0. \tag{25}$$

Thus the stress intensity factor remains constant after the start of crack propagation until the arrival of the first reflected stress wave at time $t = t_1$. The stress ahead of the reflected image of the crack tip is given from eqn (12) as

$$\tau_2(\eta_0, \xi_0) = \frac{1}{\pi(\xi_0-\xi_i)^{1/2}}\int_{\xi'}^{\xi_i}\frac{K_0}{(2\pi)^{1/2}}\frac{1}{(\xi-\xi')^{1/2}}\frac{(\xi_i-\xi)^{1/2}}{(\xi_0-\xi)}\,\mathrm{d}\xi \tag{26}$$

which gives

$$\tau_2(\eta_0, \xi_0) = \frac{K_0}{(2\pi)^{1/2}}\left[\frac{1}{(\xi_0-\xi_i)^{1/2}} - \frac{1}{(\xi_0-\xi')^{1/2}}\right] \tag{27}$$

where (η_0, ξ_i) and (η_0, ξ') are the present position and the original position of the reflected image of the crack tip along the line $\eta = \eta_0$, where $\xi' = \eta_0 - b\sqrt{2}$. Thus for times $t_1 < t < t_2$, where

$$t_2 = \frac{b}{c}\left(\frac{2}{1+v/c}\right)^2 \tag{28}$$

is the time at which a stress wave emitted from the crack tip at the start of crack propagation has been reflected at the specimen surface, diffracted at the crack tip, reflected again at the uncracked surface and arrives at the crack tip, the stress intensity factor is given from eqn (16) as

$$K_d = (1 - v/c)^{1/2}\left(\frac{2}{\pi}\right)^{1/2}\frac{K_0}{(2\pi)^{1/2}}\left\{\int_{\eta'}^{\eta_r}\frac{1}{(\eta-\eta')^{1/2}(\eta_r-\eta)^{1/2}}\,d\eta\right.$$
$$\left.+\int_{b/\sqrt{2}}^{\eta_r}\left[\frac{1}{[\xi_0-\xi_i(\eta)]^{1/2}(\eta_r-\eta)^{1/2}}-\frac{1}{[\xi_0-\xi'(\eta)]^{1/2}(\eta_r-\eta)^{1/2}}\right]d\eta\right\} \tag{29}$$

where $\eta_r(\xi_0)$ is the instantaneous position of the crack tip (point P_2 on Fig. 4), and

$$\xi_i(\eta) = \left(\frac{1+\beta}{1-\beta}\right)\eta - \frac{b\sqrt{2}}{(1-\beta)} \tag{30}$$

$$\xi'(\eta) = \eta - b\sqrt{2} \tag{31}$$

where $\beta = v/c$. Substitution for these values for $\xi_i(\eta)$ and $\xi'(\eta)$ in eqn (29) and integration then gives the stress intensity factor as

$$K_d = (1-\beta)^{1/2}K_0\left\{1+\frac{2}{\pi}\left(\frac{1-\beta}{1+\beta}\right)^{1/2}\ln\left[\frac{[4b-4\beta s]^{1/2}}{(1-\beta)^{1/2}[(1-\beta)s+2b]^{1/2}-(1+\beta)^{1/2}[(1+\beta)s-2b]^{1/2}}\right]\right.$$
$$\left.-\frac{2}{\pi}\ln\left[\frac{[4b-2\beta s]^{1/2}}{[(1-\beta)s+2b]^{1/2}-[(1+\beta)s-2b]^{1/2}}\right]\right\}. \tag{32}$$

This is shown as a function of time in Fig. 5(a) and as a function of the instantaneous crack position in Fig. 5(b). The short vertical lines show the time of arrival of the first reflected stress waves, and the stress intensity factor is shown for times up to t_2. These figures show that the stress intensity factor starts to increase gradually after the arrival at the crack tip of the first stress wave reflected from the uncracked surface of the specimen. However, the increase in the stress intensity factor is not great.

The same technique is used to calculate the effect of other contributions to $p(u)$ on the dynamic stress intensity factor. Applying this to the next order term

$$p(u) = \frac{A_1}{(2\pi)^{1/2}} \tag{33}$$

this gives the contribution to the dynamic stress intensity factor for times before the arrival of the first reflected wave, i.e. for times less than t_1, as

$$K_d = (1 - v/c)^{1/2}A_1\frac{vt}{2\sqrt{2}} \tag{34}$$

and the contribution to the stress ahead of the reflected image of the crack tip as

$$\tau'(\eta_0, \xi_0) = \frac{A_1}{(2\pi)^{1/2}}\left[\frac{1}{2}(\xi_0-\xi_i)^{1/2}-(\xi_0-\xi')^{1/2}+\frac{1}{2}\frac{(\xi_0-\xi')}{(\xi_0-\xi_i)^{1/2}}\right] \tag{35}$$

where $\xi_i(\eta)$ and $\xi'(\eta)$ are given in eqns (30) and (31). Likewise the contribution to the dynamic stress intensity factor for times between t_1 and t_2 is given by

$$K_d = (1-\beta)^{1/2}\frac{A_1}{2\sqrt{2}}\left\{\beta s-\frac{2}{\pi}\frac{\beta}{(1+\beta)}[(1-\beta)s+2b]^{1/2}[(1+\beta)s-2b]^{1/2}\right.$$
$$+\frac{2}{\pi}\left(\frac{1-\beta}{1+\beta}\right)^{1/2}\left[(4b-2\beta s)+\frac{\beta}{(1-\beta^2)}(4b-4\beta s)\right]$$

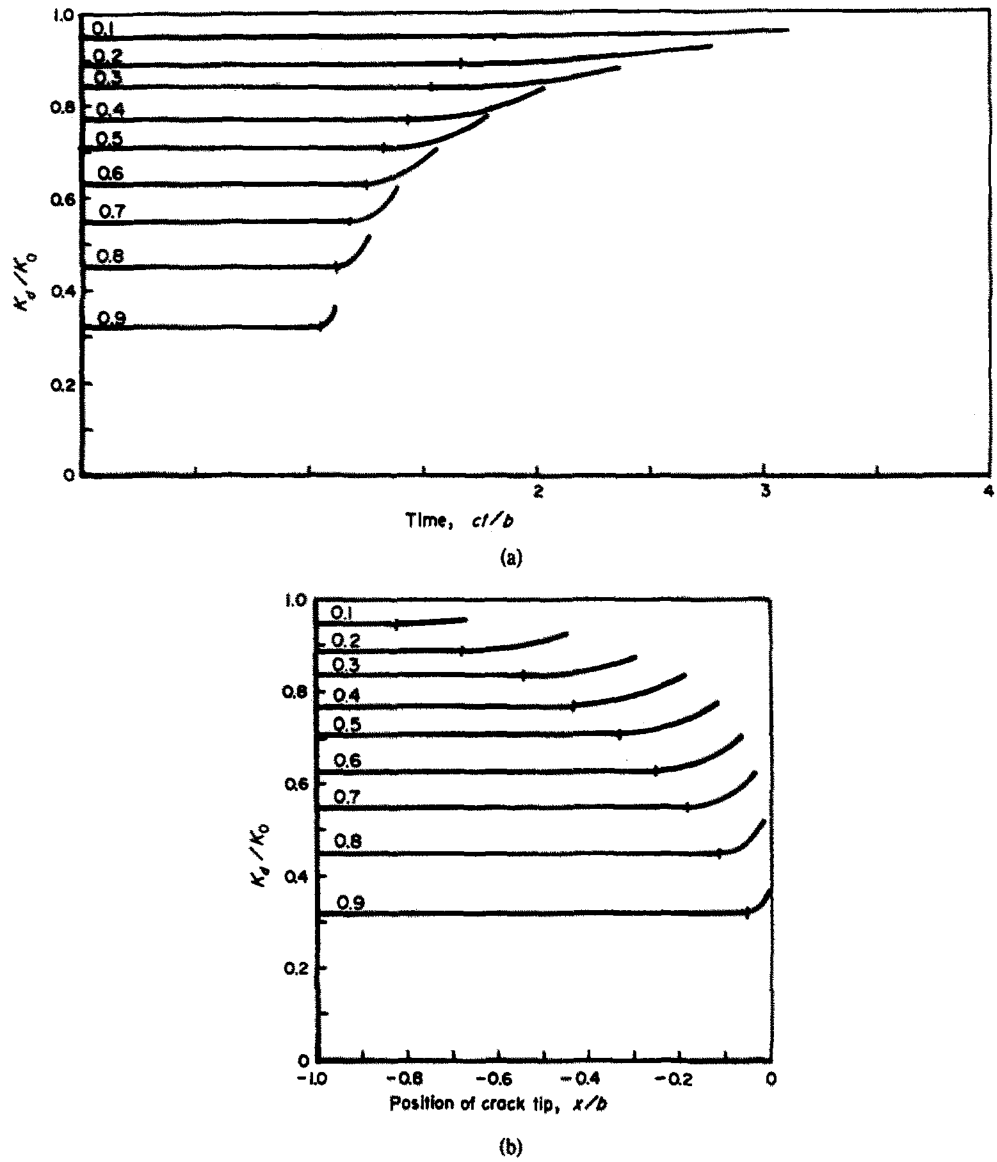

Fig. 5. (a) Variation of dynamic stress intensity factor with time after the start of crack propagation for the different crack velocities v/c shown. (b) Variation of dynamic stress intensity factor with position of crack tip for the different crack velocities v/c shown.

$$\times \ln\left[\frac{[4b-4\beta s]^{1/2}}{(1-\beta)^{1/2}[(1-\beta)s+2b]^{1/2}-(1+\beta)^{1/2}[(1+\beta)s-2b]^{1/2}}\right]$$

$$\left.-\frac{2}{\pi}(4b-2\beta s)\ln\left[\frac{[4b-2\beta s]^{1/2}}{[(1-\beta)s+2b]^{1/2}-[(1+\beta)s-2b]^{1/2}}\right]\right\} \tag{36}$$

The dynamic stress intensity factor for a particular form of $p(u)$ is then found by substituting for the appropriate values of K_0 and A_1 and adding the contributions from eqns (25) and (34) and eqns (32) and (36). The effect of the first reflections for both contributions to the dynamic stress intensity factor is shown in Fig. 6, where the dynamic stress intensity factor is given in the form

$$k_1 = \frac{K_d}{K_0(1-\beta)^{1/2}} \tag{37}$$

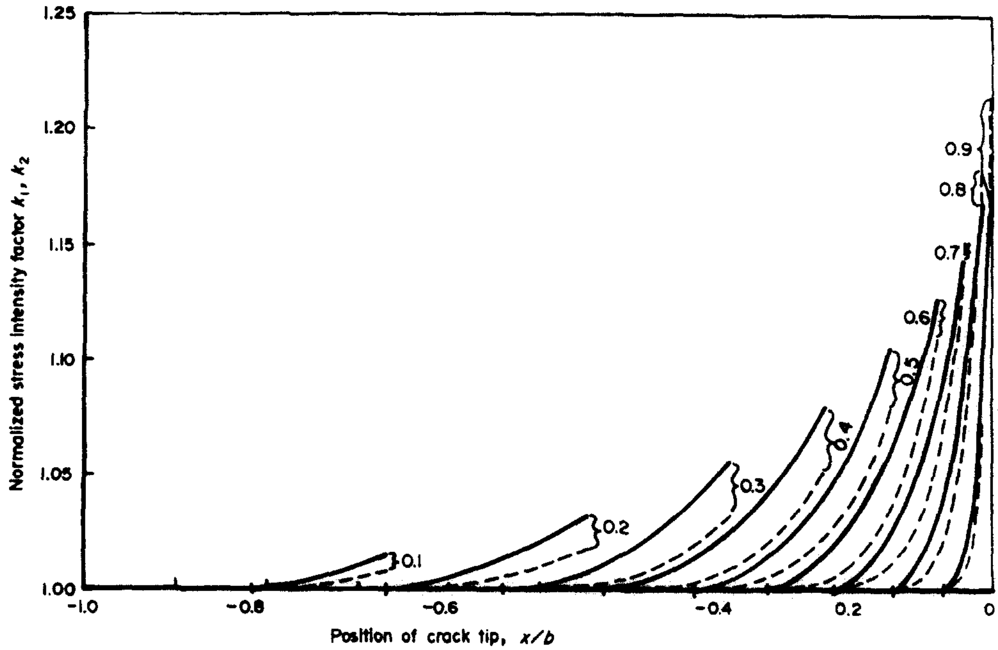

Fig. 6. Effect of the first reflected stress wave on the dynamic stress intensity factor for the different values of v/c shown. The dynamic stress intensity factors are given in the normalized forms of eqns (37) and (38) so that k_1 (full lines) and k_2 (dashed lines) give the contributions from the $u^{-1/2}$ and $u^{+1/2}$ terms for the stress ahead of the original crack tip respectively.

for the controbution from the $u^{-1/2}$ term in $p(u)$ and in the form

$$k_2 = \frac{K_d}{A_1(1-\beta)^{1/2}} \frac{2\sqrt{2}}{\beta s} \tag{38}$$

for the $u^{+1/2}$ term. In this way the curves show the percentage change in the dynamic stress intensity factor due to the arrival of the reflected wave, compared with the result ($k_1 = 1$, $k_2 = 1$) that would be obtained in the absence of the reflections. Both the contributions give a similar small increase in the dynamic stress intensity factor after the arrival of the reflected wave, the percentage change for the $u^{+1/2}$ term in $p(u)$ being smaller than that for the $u^{-1/2}$ term except for crack velocities very close to the velocity c of the shear waves. Thus in general, unless the dynamic stress intensity factor were to become negative, the arrival of the first stress wave reflected from the uncracked surface of the specimen will be accompanied by a small gradual increase in the dynamic stress intensity factor.

As is shown in Figs. 5(b) and 6, even for quite moderate crack velocities (i.e. $v/c \geq 0.2$) these solutions are valid for crack propagation across a substantial proportion of remaining width of the specimen that was not cracked before the start of dynamic crack propagation. The solution may be extended by calculating the effect of subsequent stress wave reflections.

5. CONCLUSIONS

1. Kostrov's analysis[1] for the effect of reflected stress waves on the dynamic stress intensity factor for a propagating mode III crack cannot be applied directly to a crack in a finite specimen, where stress waves are reflected from the uncracked surface of the specimen.

2. Solutions to the problem of crack propagation in a finite specimen may be obtained through modified use of Kostrov's analysis by considering the effect of the stress waves reflected from the uncracked surface separately and treating these as incident waves, following a related analysis due to Achenbach[8–10].

3. In this way solutions may in principle be obtained for any time after the start of crack propagation, although as in any analysis of the Kostrov type the integrals become increasingly complicated as the number of reflections increases.

4. The method of solution has been illustrated by a simple example, in which it is shown that the arrival of the first reflected stress waves from the uncracked surface cause a small gradual increase in the dynamic stress intensity factor.

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